

# RISING JUNIOR SUMMER PACKET part 2 ALGEBRA

1. [5 marks]

Find the value of the constant term in the expansion of  $x^4 \left(x + \frac{3}{x^2}\right)^5$ .

2. [4 marks]

Boxes of mixed fruit are on sale at a local supermarket.

Box A contains 2 bananas, 3 kiwifruit and 4 melons, and costs \$6.58.

Box B contains 5 bananas, 2 kiwifruit and 8 melons and costs \$12.32.

Box C contains 5 bananas and 4 kiwifruit and costs \$3.00.

Find the cost of each type of fruit.

3. [6 marks]

The coefficient of  $x^2$  in the expansion of  $\left(\frac{1}{x} + 5x\right)^8$  is equal to the coefficient of  $x^4$  in the expansion of  $(a + 5x)^7$ ,  $a \in \mathbb{R}$ . Find the value of  $a$ .

4. [5 marks]

Given that  $\log_{10} \left(\frac{1}{2\sqrt{2}}(p + 2q)\right) = \frac{1}{2}(\log_{10} p + \log_{10} q)$ ,  $p > 0$ ,  $q > 0$ , find  $p$  in terms of  $q$ .

5. [6 marks]

In a trial examination session a candidate at a school has to take 18 examination papers including the physics paper, the chemistry paper and the biology paper. No two of these three papers may be taken consecutively. There is no restriction on the order in which the other examination papers may be taken.

Find the number of different orders in which these 18 examination papers may be taken.

6. [5 marks]

Find the constant term in the expansion of  $\left(4x^2 - \frac{3}{2x}\right)^{12}$ .

7. [6 marks] Solve the simultaneous equations

$$\ln \frac{y}{x} = 2$$

$$\ln x^2 + \ln y^3 = 7.$$

8. [6 marks]

Find the coefficient of  $x^{-2}$  in the expansion of  $(x - 1)^3 \left(\frac{1}{x} + 2x\right)^6$ .

9. [6 marks]

The sum of the second and third terms of a geometric sequence is 96.

The sum to infinity of this sequence is 500.

Find the possible values for the common ratio,  $r$ .

10. [7 marks]

The fourth term in an arithmetic sequence is 34 and the tenth term is 76.

(a) Find the first term and the common difference.

(b) The sum of the first  $n$  terms exceeds 5000. Find the least possible value of  $n$ .

11. [7 marks]

Find the constant term in the expansion of  $\left(x - \frac{2}{x}\right)^4 \left(x^2 + \frac{2}{x}\right)^3$ .

12. [6 marks]

A complex number  $z$  is given by  $z = \frac{a+i}{a-i}$ ,  $a \in \mathbb{R}$ .

(a) Determine the set of values of  $a$  such that

(i)  $z$  is real;

(ii)  $z$  is purely imaginary.

(b) Show that  $|z|$  is constant for all values of  $a$ .

13a. [4 marks]

The 3rd term of an arithmetic sequence is 1407 and the 10th term is 1183.

Find the first term and the common difference of the sequence.

14. [3 marks] Express the binomial coefficient  $\binom{3n+1}{3n-2}$  as a polynomial in  $n$ .

15a. [3 marks]

Consider a geometric sequence with a first term of 4 and a fourth term of  $-2.916$ .

Find the common ratio of this sequence.

15b. [2 marks]

Find the sum to infinity of this sequence.

16a. [3 marks]

The seventh, third and first terms of an arithmetic sequence form the first three terms of a geometric sequence.

The arithmetic sequence has first term  $a$  and non-zero common difference  $d$ . Show that  $d = \frac{a}{2}$ .

16b. [6 marks]

The seventh term of the arithmetic sequence is 3. The sum of the first  $n$  terms in the arithmetic sequence exceeds the sum of the first  $n$  terms in the geometric sequence by at least 200.

Find the least value of  $n$  for which this occurs.

17a. [4 marks]

Find the term in  $x^5$  in the expansion of  $(3x + A)(2x + B)^6$ .

17b. [4 marks]

Mina and Norbert each have a fair cubical die with faces labelled 1, 2, 3, 4, 5 and 6; they throw it to decide if they are going to eat a cookie.

Mina throws her die just once and she eats a cookie if she throws a four, a five or a six.

Norbert throws his die six times and each time eats a cookie if he throws a five or a six.

Calculate the probability that five cookies are eaten.

18. [4 marks]

Find the sum of all the multiples of 3 between 100 and 500.

19. [6 marks]

A metal rod 1 metre long is cut into 10 pieces, the lengths of which form a geometric sequence. The length of the longest piece is 8 times the length of the shortest piece. Find, to the nearest millimetre, the length of the shortest piece.

20a. [4 marks]

The sum of the first 16 terms of an arithmetic sequence is 212 and the fifth term is 8.

Find the first term and the common difference.

20b. [3 marks]

Find the smallest value of  $n$  such that the sum of the first  $n$  terms is greater than 600.

21a. [2 marks]

Each time a ball bounces, it reaches 95 % of the height reached on the previous bounce.

Initially, it is dropped from a height of 4 metres.

What height does the ball reach after its fourth bounce?

21b. [3 marks]

How many times does the ball bounce before it no longer reaches a height of 1 metre?

21c. [3 marks]

What is the total distance travelled by the ball?

22. [7 marks]

Given that  $z = \frac{2-i}{1+i} - \frac{6+8i}{u+i}$ , find the values of  $u$ ,  $u \in \mathbb{R}$ , such that  $\operatorname{Re} z = \operatorname{Im} z$ .

23a. [1 mark]

Write down the quadratic expression  $2x^2 + x - 3$  as the product of two linear factors.

23b. [4 marks]

Hence, or otherwise, find the coefficient of  $x$  in the expansion of  $(2x^2 + x - 3)^8$ .

24. [6 marks] Solve the following system of equations.

$$\log_{x+1} y = 2$$

$$\log_{y+1} x = \frac{1}{4}$$