RISING JUNIOR SUMMER PACKET part 1 TRIGONOMETRY

1. *[7 marks]*

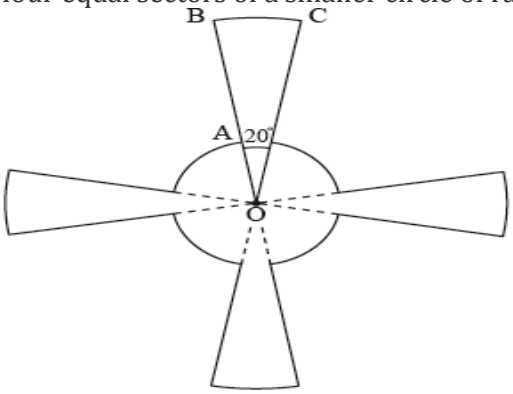
Consider triangle ABC with $\mathrm{B\hat{A}C} = 37.8°$, AB = 8.75 and BC = 6 .Find AC.

2. *[5 marks]*

Let $f(x) = \tan(x + \pi)\cos\left(x - \frac{\pi}{2}\right)$ where $0 < x < \frac{\pi}{2}$. Express $f(x)$ in terms of sin x and cos x.

3. *[4 marks]*

This diagram shows a metallic pendant made out of four equal sectors of a larger circle of radius $\mathrm{OB} = 9\ \mathrm{cm}$ and four equal sectors of a smaller circle of radius $\mathrm{OA} = 3\ \mathrm{cm}$. The angle $\mathrm{BOC} = 20°$.



Find the area of the pendant.

4. *[5 marks]*

ABCD is a quadrilateral where $\mathrm{AB} = 6.5,\ \mathrm{BC} = 9.1,\ \mathrm{CD} = 10.4,\ \mathrm{DA} = 7.8$ and $\mathrm{C\hat{D}A} = 90°$. Find $\mathrm{A\hat{B}C}$, giving your answer correct to the nearest degree.

5. *[6 marks]*

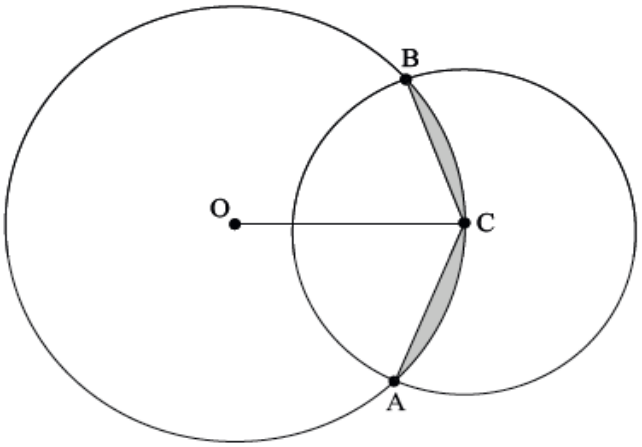
Triangle ABC has area $21\ \mathrm{cm}^2$. The sides AB and AC have lengths 6 cm and 11 cm respectively. Find the two possible lengths of the side BC.

6. *[6 marks]*

A triangle ABC has $\hat{A} = 50°$, $\mathrm{AB} = 7\ \mathrm{cm}$ and $\mathrm{BC} = 6\ \mathrm{cm}$. Find the area of the triangle given that it is smaller than $10\ \mathrm{cm}^2$.

7. *[6 marks]*

The following diagram shows two intersecting circles of radii 4 cm and 3 cm. The centre C of the smaller circle lies on the circumference of the bigger circle. O is the centre of the bigger circle and the two circles intersect at points A and B.



Find:

(a) $\mathrm{B\hat{O}C}$;

(b) the area of the shaded region.

8. *[6 marks]*

The vertices of an equilateral triangle, with perimeter *P* and area *A* , lie on a circle with radius *r* . Find an expression for $\frac{P}{A}$ in the form $\frac{k}{r}$, where $k \in \mathbb{Z}^+$.

9. *[6 marks]*

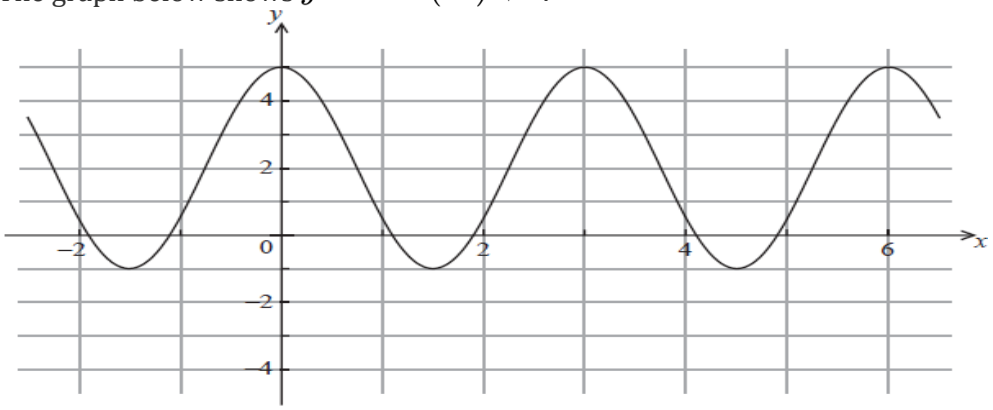
Triangle ABC has AB = 5 cm, BC = 6 cm and area 10 cm^2.

(a) Find $\sin \hat{B}$.

(b) **Hence**, find the two possible values of AC, giving your answers correct to two decimal places.

10. *[4 marks]*

The graph below shows $y = a\cos(bx) + c$.



Find the value of *a*, the value of *b* and the value of *c*.

11. *[12 marks]*

The interior of a circle of radius 2 cm is divided into an infinite number of sectors. The areas of these sectors form a geometric sequence with common ratio *k*. The angle of the first sector is θ radians.

(a) Show that $\theta = 2\pi(1 - k)$.

(b) The perimeter of the third sector is half the perimeter of the first sector.

Find the value of *k* and of θ.

12. *[6 marks]*

Consider the triangle ABC where $\text{B}\hat{\text{A}}\text{C} = 70°$, AB = 8 cm and AC = 7 cm. The point D on the side BC is such that $\frac{\text{BD}}{\text{DC}} = 2$.

Determine the length of AD.

13. *[7 marks]*

In a triangle ABC, $\hat{A} = 35°$, BC = 4 cm and AC = 6.5 cm. Find the possible values of $\hat{B}$ and the corresponding values of AB.

14. *[6 marks]*

A system of equations is given by

$$\cos x + \cos y = 1.2$$

$$\sin x + \sin y = 1.4 \text{ .}$$

(a) For each equation express *y* in terms of *x*.

(b) **Hence** solve the system for $0 < x < \pi,\ 0 < y < \pi$.

15. *[6 marks]*

The depth, *h*(*t*) metres, of water at the entrance to a harbour at *t* hours after midnight on a particular day is given by

$$h(t) = 8 + 4\sin\left(\frac{\pi t}{6}\right),\ 0 \leqslant t \leqslant 24.$$

(a) Find the maximum depth and the minimum depth of the water.

(b) Find the values of *t* for which $h(t) \geqslant 8$.

16a. *[3 marks]*

Given that $\arctan\frac{1}{2} - \arctan\frac{1}{3} = \arctan a,\ a \in \mathbb{Q}^+$, find the value of a.

16b. *[2 marks]*

Hence, or otherwise, solve the equation $\arcsin x = \arctan a$.

17a. *[5 marks]*

In a triangle ABC, $\mathrm{AB} = 4\ \mathrm{cm}$, $\mathrm{BC} = 3\ \mathrm{cm}$ and $\mathrm{B\hat{A}C} = \frac{\pi}{9}$.

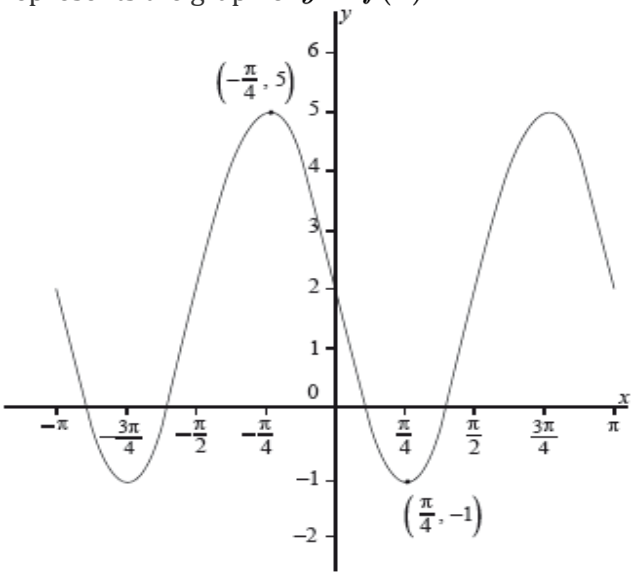
Use the cosine rule to find the two possible values for AC.

17b. *[3 marks]*

Find the difference between the areas of the two possible triangles ABC.

18a. *[4 marks]*

A function is defined by $f(x) = A\sin(Bx) + C,\ -\pi \leq x \leq \pi$, where $A,\ B,\ C \in \mathbb{Z}$. The following diagram represents the graph of $y = f(x)$.



Find the value of

(i) A;

(ii) B;

(iii) C.

18b. *[2 marks]*

Solve $f(x) = 3$ for $0 \leq x \leq \pi$.

19a. *[2 marks]*

In triangle ABC, $\mathrm{AB} = 5\ \mathrm{cm}$, $\mathrm{BC} = 12\ \mathrm{cm}$ and $\mathrm{A\hat{B}C} = 100^\circ$.

Find the area of the triangle.

19b. *[2 marks]*

Find AC.

20a. *[6 marks]*

In triangle ABC,

$3\sin B + 4\cos C = 6$ and

$4\sin C + 3\cos B = 1$.

Show that $\sin(B + C) = \frac{1}{2}$.

20b. *[5 marks]*

Robert conjectures that $\mathrm{C\hat{A}B}$ can have two possible values.

Show that Robert's conjecture is incorrect by proving that $\mathrm{C\hat{A}B}$ has only one possible value.

21a. *[3 marks]*

Solve the equation $3\cos^2 x - 8\cos x + 4 = 0$, where $0 \leqslant x \leqslant 180^\circ$, expressing your answer(s) to the nearest degree.

21b. *[3 marks]*

Find the exact values of $\sec x$ satisfying the equation $3\sec^4 x - 8\sec^2 x + 4 = 0$.

22a. *[2 marks]*

Consider a triangle ABC with $\mathrm{B\hat{A}C} = 45.7°$, AB = 9.63 cm and BC = 7.5 cm .

By drawing a diagram, show why there are two triangles consistent with this information.

22b. *[6 marks]*

Find the possible values of AC .

23a. *[3 marks]*

Consider the function f defined by $f(x) = 3x \arccos(x)$ where $-1 \leqslant x \leqslant 1$.

Sketch the graph of f indicating clearly any intercepts with the axes and the coordinates of any local maximum or minimum points.

23b. *[2 marks]*

State the range of f.

23c. *[4 marks]*

Solve the inequality $|3x \arccos(x)| > 1$.

24a. *[4 marks]*

Farmer Bill owns a rectangular field, 10 m by 4 m. Bill attaches a rope to a wooden post at one corner of his field, and attaches the other end to his goat Gruff.

Given that the rope is 5 m long, calculate the percentage of Bill's field that Gruff is able to graze. Give your answer correct to the nearest integer.

24b. *[4 marks]*

Bill replaces Gruff's rope with another, this time of length $a,\ 4 < a < 10$, so that Gruff can now graze exactly one half of Bill's field.

Show that a satisfies the equation

$$a^2 \arcsin\left(\frac{4}{a}\right) + 4\sqrt{a^2 - 16} = 40.$$

24c. *[2 marks]*

Find the value of a.

25a. *[2 marks]*

In triangle PQR, $\mathrm{PR} = 12$ cm, $\mathrm{QR} = p$ cm, $\mathrm{PQ} = r$ cm and $\mathrm{Q\hat{P}R} = 30°$.

Use the cosine rule to show that $r^2 - 12\sqrt{3}r + 144 - p^2 = 0$.

25b. *[3 marks]*

Consider the possible triangles with $\mathrm{QR} = 8$ cm.

Calculate the two corresponding values of PQ.

25c. *[3 marks]*

Hence, find the area of the smaller triangle.

25d. *[7 marks]*

Consider the case where p, the length of QR is not fixed at 8 cm.

Determine the range of values of p for which it is possible to form two triangles.