Welcome to IB Higher Level Math.

This class is reserved for the top math students that have a history of exemplary performance in prerequisite math courses and a desire to pursue a career involving advanced mathematics.

On the first day of school in junior year, you will be assessed on your readiness for the course and mastery of prerequisite knowledge of ALGEBRA and TRIGONOMETRY which you learned in 9th and 10th grade classes and mu alpha theta. The questions on the summer work quiz in August of this year (11th grade) are taken from the attached summer work packets which consist of past IB external examination questions. Please complete the summer work BEFORE August.

The course moves at a fast pace and covers advanced math topics that are not typically covered in a traditional high school math course.

On the semester ONE exam in junior year, you will be assessed on your mastery of CALCULUS. The exam in December of this year (11^{th} grade) covers most of the AP Calculus BC curriculum. As such, it is highly recommended that you consider taking the AP Calculus BC course on FLVS or taking the AP Calculus BC elective offered at Robinson if it works with your schedule. One of the many advantages of taking the BC course is that the district will pay for you to take the BC exam at the end of junior year, so that you can get the college credit for free. You will be expected to take the AP Calculus BC external exam in May (11^{th} grade).

Your second semester exam in junior year consists of all of the BC topics as well as some additional topics that are unique to IB such as Homogeneous Differential Equations, Integrating Factor and First Order Linear Differential Equations. You will also be required to begin your IB HL Math Internal Assessment on a topic of your choice.

In senior year, we will be focused mainly on Proofs (especially by math INDUCTION, CONTRADICTION), 3D VECTORS (IB Physics helps) including intersections of three dimensional planes using matrix algebra , probability and STATISTICS (AP stats helps) including Baye's Theorem and calculus applied to continuous probability density functions, Complex numbers (DeMoivre's Theorem - MAO helps) and many more interesting mathematical concepts integrated into one holistic course where the math seamlessly ties together.

This course is the most challenging course that is offered (to my knowledge). Your classmates will be students who share your true passion and love for mathematics. We eat, sleep and breathe math. We seek true knowledge, understanding and appreciation of the beauty of pure mathematics. We learn from our mistakes and celebrate the success of others. I have had the privilege of teaching this course from the very first year that it was introduced at Robinson High School and have had the pleasure of teaching the most outstanding students each year and seeing their hard work pay off year after year. It will not be easy, but it will be so worth it!

RISING JUNIOR SUMMER PACKET part 1 TRIGONOMETRY

1. *[7 marks]*

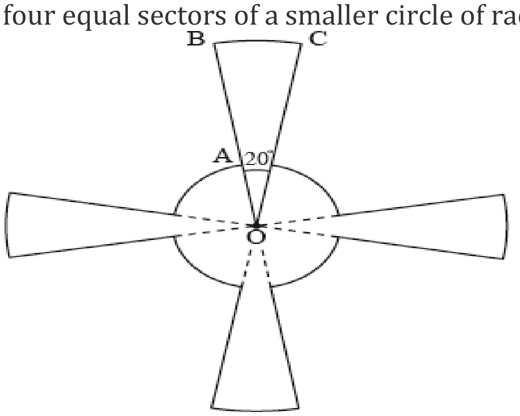
Consider triangle ABC with $\mathrm{B\hat{A}C} = 37.8°$, AB = 8.75 and BC = 6 .Find AC.

2. *[5 marks]*

Let $f(x) = \tan(x+\pi)\cos\left(x - \frac{\pi}{2}\right)$ where $0 < x < \frac{\pi}{2}$. Express $f(x)$ in terms of sin x and cos x.

3. *[4 marks]*

This diagram shows a metallic pendant made out of four equal sectors of a larger circle of radius $\mathrm{OB} = 9\ \mathrm{cm}$ and four equal sectors of a smaller circle of radius $\mathrm{OA} = 3\ \mathrm{cm}$. The angle $\mathrm{BOC} = 20°$.



Find the area of the pendant.

4. *[5 marks]*

ABCD is a quadrilateral where $\mathrm{AB} = 6.5,\ \mathrm{BC} = 9.1,\ \mathrm{CD} = 10.4,\ \mathrm{DA} = 7.8$ and $\mathrm{C\hat{D}A} = 90°$. Find $\mathrm{A\hat{B}C}$, giving your answer correct to the nearest degree.

5. *[6 marks]*

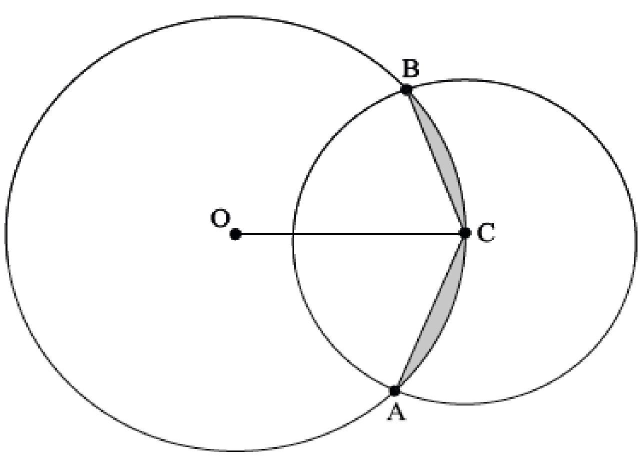
Triangle ABC has area $21\ \mathrm{cm}^2$. The sides AB and AC have lengths 6 cm and 11 cm respectively. Find the two possible lengths of the side BC.

6. *[6 marks]*

A triangle ABC has $\hat{A} = 50°$, $\mathrm{AB} = 7\ \mathrm{cm}$ and $\mathrm{BC} = 6\ \mathrm{cm}$. Find the area of the triangle given that it is smaller than $10\ \mathrm{cm}^2$.

7. *[6 marks]*

The following diagram shows two intersecting circles of radii 4 cm and 3 cm. The centre C of the smaller circle lies on the circumference of the bigger circle. O is the centre of the bigger circle and the two circles intersect at points A and B.



Find:

(a) $\mathrm{B\hat{O}C}$;
(b) the area of the shaded region.

8. *[6 marks]*

The vertices of an equilateral triangle, with perimeter *P* and area *A* , lie on a circle with radius *r* . Find an expression for $\frac{P}{A}$ in the form $\frac{k}{r}$, where $k \in \mathbb{Z}^{+}$.

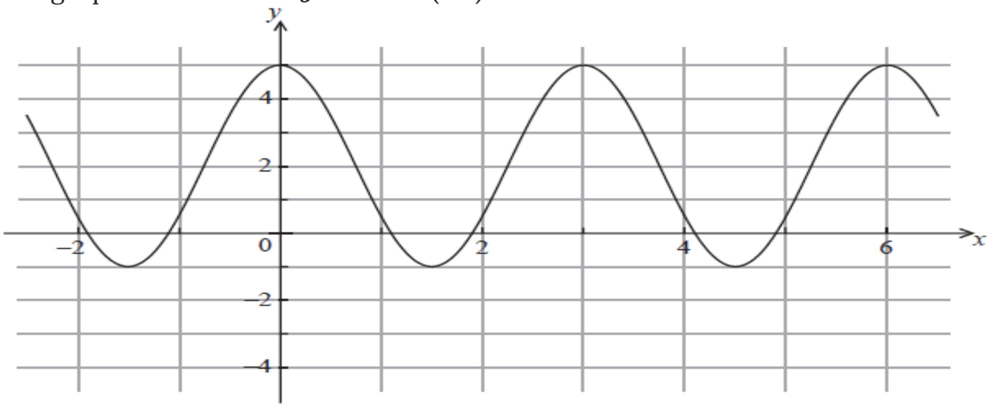
9. *[6 marks]*

Triangle ABC has AB = 5 cm, BC = 6 cm and area 10 cm^2.

(a) Find $\sin \hat{B}$.
(b) **Hence**, find the two possible values of AC, giving your answers correct to two decimal places.

10. *[4 marks]*

The graph below shows $y = a\cos(bx) + c$.



Find the value of *a*, the value of *b* and the value of *c*.

11. *[12 marks]*

The interior of a circle of radius 2 cm is divided into an infinite number of sectors. The areas of these sectors form a geometric sequence with common ratio *k*. The angle of the first sector is θ radians.

(a) Show that $\theta = 2\pi(1-k)$.
(b) The perimeter of the third sector is half the perimeter of the first sector.
Find the value of *k* and of θ.

12. *[6 marks]*

Consider the triangle ABC where $\mathrm{B\hat{A}C} = 70°$, AB = 8 cm and AC = 7 cm. The point D on the side BC is such that $\frac{\mathrm{BD}}{\mathrm{DC}} = 2$.

Determine the length of AD.

13. *[7 marks]*

In a triangle ABC, $\hat{A} = 35°$, BC = 4 cm and AC = 6.5 cm. Find the possible values of $\hat{B}$ and the corresponding values of AB.

14. *[6 marks]*

A system of equations is given by

$$\cos x + \cos y = 1.2$$

$$\sin x + \sin y = 1.4 .$$

(a) For each equation express *y* in terms of *x*.
(b) **Hence** solve the system for $0 < x < \pi,\ 0 < y < \pi$.

15. *[6 marks]*

The depth, *h*(*t*) metres, of water at the entrance to a harbour at *t* hours after midnight on a particular day is given by

$h(t) = 8 + 4\sin\left(\frac{\pi t}{6}\right),\ 0 \leqslant t \leqslant 24.$

(a) Find the maximum depth and the minimum depth of the water.

(b) Find the values of *t* for which $h(t) \geqslant 8$.

16a. *[3 marks]*

Given that $\arctan\frac{1}{2} - \arctan\frac{1}{3} = \arctan a,\ a \in \mathbb{Q}^+$, find the value of *a*.

16b. *[2 marks]*

Hence, or otherwise, solve the equation $\arcsin x = \arctan a$.

17a. *[5 marks]*

In a triangle $\text{ABC},\ \text{AB} = 4\text{ cm},\ \text{BC} = 3\text{ cm}$ and $\text{B}\hat{\text{A}}\text{C} = \frac{\pi}{9}$.

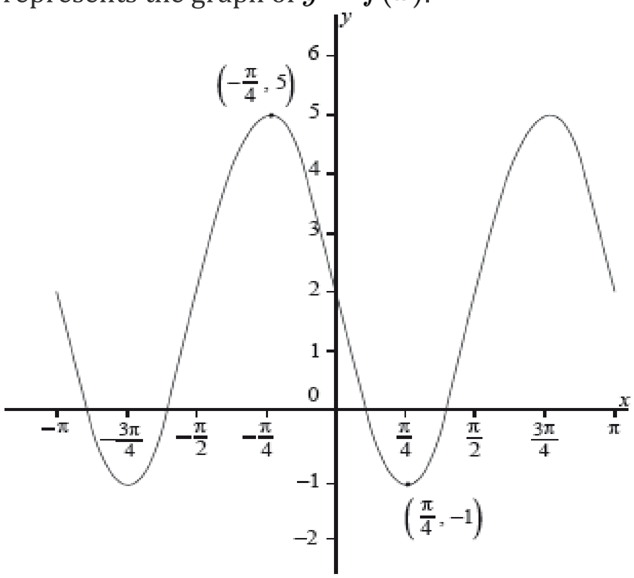
Use the cosine rule to find the two possible values for AC.

17b. *[3 marks]*

Find the difference between the areas of the two possible triangles ABC.

18a. *[4 marks]*

A function is defined by $f(x) = A\sin(Bx) + C,\ \ -\pi \le x \le \pi$, where $A,\ B,\ C \in \mathbb{Z}$. The following diagram represents the graph of $y = f(x)$.



Find the value of

(i) A;

(ii) B;

(iii) C.

18b. *[2 marks]*

Solve $f(x) = 3$ for $0 \le x \le \pi$.

19a. *[2 marks]*

In triangle ABC, $\text{AB} = 5\text{ cm}$, $\text{BC} = 12\text{ cm}$ and $\text{A}\hat{\text{B}}\text{C} = 100°$.

Find the area of the triangle.

19b. *[2 marks]*

Find AC.

20a. *[6 marks]*

In triangle ABC,

$3\sin B + 4\cos C = 6$ and

$4\sin C + 3\cos B = 1$.

Show that $\sin(B + C) = \frac{1}{2}$.

20b. *[5 marks]*

Robert conjectures that $\hat{CAB}$ can have two possible values.

Show that Robert's conjecture is incorrect by proving that $\hat{CAB}$ has only one possible value.

21a. *[3 marks]*

Solve the equation $3\cos^2 x - 8\cos x + 4 = 0$, where $0 \leqslant x \leqslant 180^\circ$, expressing your answer(s) to the nearest degree.

21b. *[3 marks]*

Find the exact values of $\sec x$ satisfying the equation $3\sec^4 x - 8\sec^2 x + 4 = 0$.

22a. *[2 marks]*

Consider a triangle ABC with $\hat{BAC} = 45.7^\circ$, AB = 9.63 cm and BC = 7.5 cm .

By drawing a diagram, show why there are two triangles consistent with this information.

22b. *[6 marks]*

Find the possible values of AC .

23a. *[3 marks]*

Consider the function f defined by $f(x) = 3x \arccos(x)$ where $-1 \leqslant x \leqslant 1$.

Sketch the graph of f indicating clearly any intercepts with the axes and the coordinates of any local maximum or minimum points.

23b. *[2 marks]*

State the range of f.

23c. *[4 marks]*

Solve the inequality $|3x \arccos(x)| > 1$.

24a. *[4 marks]*

Farmer Bill owns a rectangular field, 10 m by 4 m. Bill attaches a rope to a wooden post at one corner of his field, and attaches the other end to his goat Gruff.

Given that the rope is 5 m long, calculate the percentage of Bill's field that Gruff is able to graze. Give your answer correct to the nearest integer.

24b. *[4 marks]*

Bill replaces Gruff's rope with another, this time of length $a,\ 4 < a < 10$, so that Gruff can now graze exactly one half of Bill's field.

Show that a satisfies the equation

$$a^2 \arcsin\left(\frac{4}{a}\right) + 4\sqrt{a^2 - 16} = 40.$$

24c. *[2 marks]*

Find the value of a.

25a. *[2 marks]*

In triangle $\mathrm{PQR},\ \mathrm{PR} = 12\ \mathrm{cm},\ \mathrm{QR} = p\ \mathrm{cm},\ \mathrm{PQ} = r\ \mathrm{cm}$ and $\hat{QPR} = 30^\circ$.

Use the cosine rule to show that $r^2 - 12\sqrt{3}r + 144 - p^2 = 0$.

25b. *[3 marks]*

Consider the possible triangles with $\mathrm{QR} = 8\ \mathrm{cm}$.

Calculate the two corresponding values of PQ.

25c. *[3 marks]*

Hence, find the area of the smaller triangle.

25d. *[7 marks]*

Consider the case where p, the length of QR is not fixed at 8 cm.

Determine the range of values of p for which it is possible to form two triangles.

RISING JUNIOR SUMMER PACKET part 2 ALGEBRA

1. *[5 marks]*

Find the value of the constant term in the expansion of $x^4\left(x+\frac{3}{x^2}\right)^5$.

2. *[4 marks]*

Boxes of mixed fruit are on sale at a local supermarket.
Box A contains 2 bananas, 3 kiwifruit and 4 melons, and costs $6.58.
Box B contains 5 bananas, 2 kiwifruit and 8 melons and costs $12.32.
Box C contains 5 bananas and 4 kiwifruit and costs $3.00.
Find the cost of each type of fruit.

3. *[6 marks]*

The coefficient of x^2 in the expansion of $\left(\frac{1}{x}+5x\right)^8$ is equal to the coefficient of x^4 in the expansion of $(a+5x)^7,\ a\in\mathbb{R}$. Find the value of a.

4. *[5 marks]*

Given that $\log_{10}\left(\frac{1}{2\sqrt{2}}(p+2q)\right)=\frac{1}{2}(\log_{10}p+\log_{10}q)\ ,\ p>0,\ q>0$, find p in terms of q.

5. *[6 marks]*

In a trial examination session a candidate at a school has to take 18 examination papers including the physics paper, the chemistry paper and the biology paper. No two of these three papers may be taken consecutively. There is no restriction on the order in which the other examination papers may be taken.
Find the number of different orders in which these 18 examination papers may be taken.

6. *[5 marks]*

Find the constant term in the expansion of $\left(4x^2-\frac{3}{2x}\right)^{12}$.

7. *[6 marks]* Solve the simultaneous equations

$$\ln\frac{y}{x}=2$$
$$\ln x^2+\ln y^3=7.$$

8. *[6 marks]*

Find the coefficient of x^{-2} in the expansion of $(x-1)^3\left(\frac{1}{x}+2x\right)^6$.

9. *[6 marks]*

The sum of the second and third terms of a geometric sequence is 96.
The sum to infinity of this sequence is 500.
Find the possible values for the common ratio, r.

10. *[7 marks]*

The fourth term in an arithmetic sequence is 34 and the tenth term is 76.

(a) Find the first term and the common difference.

(b) The sum of the first *n* terms exceeds 5000. Find the least possible value of *n*.

11. *[7 marks]*

Find the constant term in the expansion of $\left(x-\frac{2}{x}\right)^4\left(x^2+\frac{2}{x}\right)^3$.

12. *[6 marks]*

A complex number *z* is given by $z=\frac{a+\mathrm{i}}{a-\mathrm{i}},\ a\in\mathbb{R}$.

(a) Determine the set of values of *a* such that

(i) *z* is real;

(ii) *z* is purely imaginary.

(b) Show that $|z|$ is constant for all values of *a*.

13a. *[4 marks]*

The 3rd term of an arithmetic sequence is 1407 and the 10th term is 1183.
Find the first term and the common difference of the sequence.

14. *[3 marks]* Express the binomial coefficient $\binom{3n+1}{3n-2}$ as a polynomial in n.

15a. *[3 marks]*

Consider a geometric sequence with a first term of 4 and a fourth term of −2.916.
Find the common ratio of this sequence.

15b. *[2 marks]*

Find the sum to infinity of this sequence.

16a. *[3 marks]*

The seventh, third and first terms of an arithmetic sequence form the first three terms of a geometric sequence.
The arithmetic sequence has first term a and non-zero common difference d. Show that $d = \frac{a}{2}$.

16b. *[6 marks]*

The seventh term of the arithmetic sequence is 3. The sum of the first n terms in the arithmetic sequence exceeds the sum of the first n terms in the geometric sequence by at least 200.
Find the least value of n for which this occurs.

17a. *[4 marks]*

Find the term in x^5 in the expansion of $(3x + A)(2x + B)^6$.

17b. *[4 marks]*

Mina and Norbert each have a fair cubical die with faces labelled 1, 2, 3, 4, 5 and 6; they throw it to decide if they are going to eat a cookie.
Mina throws her die just once and she eats a cookie if she throws a four, a five or a six.
Norbert throws his die six times and each time eats a cookie if he throws a five or a six.
Calculate the probability that five cookies are eaten.

18. *[4 marks]*

Find the sum of all the multiples of 3 between 100 and 500.

19. *[6 marks]*

A metal rod 1 metre long is cut into 10 pieces, the lengths of which form a geometric sequence. The length of the longest piece is 8 times the length of the shortest piece. Find, to the nearest millimetre, the length of the shortest piece.

20a. *[4 marks]*

The sum of the first 16 terms of an arithmetic sequence is 212 and the fifth term is 8.
Find the first term and the common difference.

20b. *[3 marks]*

Find the smallest value of *n* such that the sum of the first *n* terms is greater than 600.

21a. *[2 marks]*

Each time a ball bounces, it reaches 95 % of the height reached on the previous bounce.
Initially, it is dropped from a height of 4 metres.
What height does the ball reach after its fourth bounce?

21b. *[3 marks]*

How many times does the ball bounce before it no longer reaches a height of 1 metre?

21c. *[3 marks]*

What is the total distance travelled by the ball?

22. *[7 marks]*

Given that $z = \frac{2-\mathrm{i}}{1+\mathrm{i}} - \frac{6+8\mathrm{i}}{u+\mathrm{i}}$, find the values of u, $u \in \mathbb{R}$, such that $\operatorname{Re} z = \operatorname{Im} z$.

23a. *[1 mark]*

Write down the quadratic expression $2x^2 + x - 3$ as the product of two linear factors.

23b. *[4 marks]*

Hence, or otherwise, find the coefficient of x in the expansion of $\left(2x^2 + x - 3\right)^8$.

24. *[6 marks]* Solve the following system of equations.

$$\log_{x+1} y = 2$$
$$\log_{y+1} x = \frac{1}{4}$$